2021

Full Marks - 80

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer ***all*** questions

1. a) Let $f(xy) = \sqrt{x^4 + y^4 + 1}$

Find $f_x(1, 2)$ and $f_y(1, 2)$ 10

b) i) Evaluate

$$\lim_{(xy)\to(0,0)} \frac{x^2 - 2xy + y^2}{x - y}$$ 3

ii) Show that

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (xy) \neq (0,0) \\ 0 & (xy) = (0,0) \end{cases}$$

is discontinuous at (0, 0). 3

OR

c) Let $f(x, y) \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & (xy) \neq (0,0) \\ 0 & (xy) = (0,0) \end{cases}$

show that f is not differentiable at (0, 0). 10

d) i) Evaluate

$$\lim_{(xy)\to(1,2)} \frac{(x^2-1)(y^2-4)}{(x-1)(y-2)}$$ 3

ii) If $f(x, y) = \cos xy^2$, show that $f_{xy} = f_{yx}$. 3

2. a) If $f(xy) = x^3 + y^3 + 3x^2 - 18y^2 + 81y + 5$

Find critical points and classify each point as a relative maximum, a relative minimum or a saddle point. 10

b) i) Find the equation of Tangent plane to the surface $yz - zx - xy + 5 = 0$ at $(1, -1, 2)$. 3

ii) Find the point on the plane $2x - y + 2z = 20$ that is nearest to the point (0, 0, 0). 3

OR

c) Find all points in the plane $x + 2y + 3z = 4$ in the first octant where $f(x, y, z) = x^2yz^3$ has a maximum value. 10

d) i) Check whether (4, 1) is a point of absolute maxima or obsolute minima, for the function $f(x, y) = x^2 + y^2 - 8x - 2y + 18$. 3

ii) Define normal of gradient. 3

3. a) Evaluate the integral

$$\int_0^2 \int_0^1 (x^2 + xy + y^2)dydx$$ .10

b) i) Change the double integral $\iint_R f(xy)dx\,dy$ to its polar integral. 3

ii) Evaluate $\iint_R (2x+3y)dx\,dy$ over

$R : 0 \le x \le 1, \quad 0 \le y \le 2.$ 3

OR

c) Evaluate the given integral by converting to polar coordinates : 10

$$\int_0^3 \int_0^{\sqrt{9-x^2}} x \, dy \, dx.$$

d) i) Change the order of integration

$$\int_0^1 dx \int_x^{\sqrt{x}} f(x, y) \, dy$$ 3

ii) Evaluate

$$\iint_R (x^2 + y^2 + 1) \, dA,$$

where D is the region inside the circle $x^2 + y^2 = 4$. 3

4. a) Show that

$$\text{grad}\left(\vec{f} \cdot \vec{g}\right) = \vec{f} \times \text{curl}\, \vec{g} + \vec{g} \times \text{curl}\, \vec{f} + \left(\vec{f} \cdot \nabla\right)\vec{g} + \left(\vec{g} \cdot \nabla\right)\vec{f}$$ 10

b) i) Compute

$$\nabla \cdot \vec{f} \text{ if } \vec{f} = \frac{\vec{r}}{r^3}$$ 3

ii) Find div F given that $F = \nabla f$ where $f(x, y, z) = x^2yz^3$. 3

OR

c) Show that 10

$$\text{Curl}\left(\vec{f} \times \vec{g}\right) = \vec{f} \,\text{div}\, \vec{g} - \vec{g} \,\text{div}\, \vec{f} + \left(\vec{g} \cdot \nabla\right)\vec{f} - \left(\vec{f} \cdot \nabla\right)\vec{g}$$

d) i) Find work done by the force

$$F = 2xy\hat{i} + (x^2 + 2)\hat{j} + y\hat{k}$$

and C is the live segment from (1, 0, 2) to (3, 4, 1). 3

ii) Evaluate

$\int_C (x^2 + y^2)\, dy$, where C is the curke

$x(t) = at^2,\ y(t) = 2at,\ 0 \le t \le 1.$ 3

5. a) Using Stoke's theorem evaluate $\iint_S (\text{curl}\, F.N)\, ds$, for the vector field $F = 2z\hat{i} + 3x\hat{j} + 5y\hat{k}$ and S is the part of paraboloid $Z = 4 - x^2 - y^2$ with $z \geq 0$. 10

b) i) State Green's theorem in Cartesian form and write its physical interpretation. 3

ii) Find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 3

OR

c) Compute

$\int_c F.dR$ where $f = \langle x^2 - y^2, y^2 - z^2, z^2 - x^2 \rangle$ and C is the triangle (1, 0, 0), (0, 1, 0) and (0, 0, 1). 10

d) i) Apply Gauss divergence theorem to evaluate $\iint_c \frac{\hat{n} \cdot \vec{r}}{r^3} ds$, if the origin lies out side S. 3

ii) Gauss divergence theorem relates : 3

a) surface and volume integral

b) line and volume integral

c) line and surface integral

d) all of these

Choose the correct option.

2020

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Answer ***all*** questions

1. a) If $\{E_n, n \geq 1\}$ is either an increasing or a decreasing sequence of events, then

$$\lim_{n\to\infty} P(E_n) = P\left(\lim_{n\to\infty}(E_n)\right)$$ 10

b) i) State and prove De-Morgan's Laws in probability. 3

ii) What are the axioms of probability. 3

OR

c) For any random variable, prove that 10

$$P(X = x) = F_x(x) - F_x(x-) \text{ for all } x \in R$$

where $F_x(x-) = \lim_{z\to x} F_x(z)$

d) i) If 3 balls drawn randomly from a bag containing 6 white and 5 black balls, what is the probability that one of the balls is white and other two are black 3

ii) How many subsets are there of a set consisting of n elements ? 3

2. a) Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$ for 10

$$f(x) = 6x(1 - x),\ 0 < x < 1$$
$$0,\quad \text{otherwise}$$

and also for

$$p(x) = \left(\frac{1}{2}\right)^{x},\ x = 1, 2, 3, \ldots..$$
$$0,\quad \text{elsewhere}$$

b) i) Define mathematical expectation.

If X have the pdf $f(x) = 2x,\ 0 < x < 1$

0, therwise

Find $E\left(\frac{1}{X}\right)$ 3

ii) If X be a random variable with a pdf f(x) and mgf M(t); f is symmetric about 0, then show that M(–t) = M(t). 3

OR

c) Find the moments of the distribution that has mgf $M(t) = (1 - t)^{-3}$, $t < 1$ using Maclaurin's series. 10

d) Let X be a random variable with pdf 6

$$f(x) = \beta^{-1} e^{(-x/\beta)}, \quad 0 < x \infty$$
$$0, \qquad \text{otherwise}$$

Find mgf, mean and variance of X

3. a) Find mean and variance of geometric random variable. 10

b) i) Calcuate Var(X) if X repesents the outcome when a fair die is rolled. 3

ii) Derive $Var(x) = E(x^2) - [E(x)]^2$. 3

OR

c) Compute the expected value and variance of negative bionomial random variable with parameters r and p. 10

d) Let X_1 and Y_2 be jointly continuous random variables with probability density function $f_{x_1x_2}$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Find the joint density function of Y_1 and Y_2 in terms of $f_{x_1x_2}$. 6

4\. a) Prove that :

$$E[(Y - g(x))^2] \geq E[(Y - E(Y|X])^2]$$

Where X and Y are random variables and f(x) is the predictor. 10

b) If X and Y are independent bionomial random variables with identical parameters n and p; calculate the conditional expected value of X given that $X + Y = m$. 6

OR

c) The joint density function of X and Y is given by

$$f(x,y) = \frac{1}{y}e^{-(y+x/y)}, \; x>0, y>0$$

Find E[X], E[Y] and show that Cov (X, Y) = 1. 10

d) A fair die is rolled. Let X and Y denote the number of rolls required to obtain 6 and 5 respectively. Find E[X] and E[X | Y = 5]. 6

5. a) Compare Markov's inequality and Chebyshev's inequality. 10

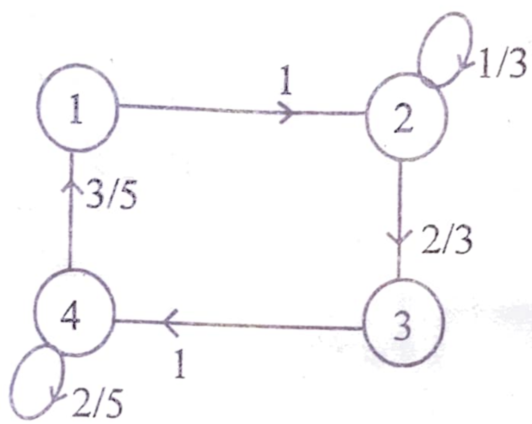
b) The number of items produced in a factory during a week is a random variable with mean 50

i) Find the probability that this week's production will exceed 75 ? 3

ii) If variance is 25, what is the probability that this week's production will be between 40 and 60 ? 3

OR

c) Consider the Markov, chain



Steady state probabilities are

$$\pi_1 = \frac{6}{31}, \pi_2 = \frac{9}{31}, \pi_3 = \frac{6}{31} \text{ and } \pi_4 = \frac{10}{31}.$$

What is the probability that the state of system resulting from transition 1000 is neither be same as the state resulting from transition 999 nor same as the state resulting from transition loop.

10

d) Prove that :

$$P_{ij}^{(n)} = \sum_{k} P_{kj}^{(n-r)} P_{ik}^{(r)} \text{ for all } 0 < r < n.$$ 6

□□

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Answer *all* questions

Part-I

1. Answer the following : 1×12

   a) What is pivotal element in LPP ?

   b) When artificial variable is required ?

   c) In system of m simultaneous Linear equation in a n unknowns and $m < n$, then the number of Basic variables will be _____.

   d) When primal-dual pair is said to be symmetric.

   e) Dual problem is to be solved by changing the objective from Minimize to maximize is true or false.

   f) What do you mean by dual price ?

   g) When TP is said to be Balanced ?

h) Every loop has odd number of cells is T or F.

i) The allocated cells in the Transpotection Table will be called ___ cells.

j) When game is said to be fair ?

k) What is zero-sum games.

l) What do you mean by value of the game.

Part-II

2. Answer any ***eight*** of the following : 2×8

a) Define slack variable with examples.

b) State the Fundamental Theorem of Duality.

c) Rewrite the LPP in standard form

$$\max z = 3x_1 + 5x_2$$

Subject to $4x_1 + x_2 \geq 11$

$$2x_1 + 5x_2 \leq 6,\ x_1 \geq 0,\ x_2 \geq 0.$$

d) What do you mean by pseudo-optimum basic feasible solution.

e) Write the Basic Duality Theorem.

f) Write the Mathematical form of Assignment problem.

g) How initial allocation for Least cost method is obtained.

h) Write the dual for general TP.

i) Is saddle point exist in the following game

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix}}$$

j) Determine the range value of p and q that will make the pay off element a_{22},

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix}}$$

Part-III

3. Answer any ***eight*** of the following : 3 × 8

a) Find one basic feasible solution to the system of Linear equation

$x_1 + 2x_2 + x_3 = 4$ and $2x_1 + x_2 + 5x_3 = 5$

b) Write the first 3 step of simplex method to solve LPP.

c) What do you mean by unbounded solution ?

d) Formulate the dual of the LPP

Maximize $z = 10x_1 + 8x_2$

Subject to constraints $x_1 + 2x_2 \geq 5$

$2x_1 - x_2 \geq 12$

$x_1 + 3x_2 \geq 4$

$x_1 \geq 0, x_2 \geq 0.$

e) Write the standard primal to LPP

Mini $z = 3x_1 - 2x_2 + 4x_3$

Subject to constraints $3x_1 + 5x_2 - 4x_3 \geq 7,$

$x_1 - 6x_2 + 3x_3 \geq 4, 7x_1 - 2x_2 + x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

f) Convert the following assignment problem to Transpotation problem.

	A_1	A_2	A_3
R_1	2	3	4
R_2	5	1	6
R_3	4	1	2

g) Define loop in TP and explain.

h) Write the first two step of NWC method.

i) Is the following two person zero-sum game is strictly determinable

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}}$$

j) What do you mean by pay off matrix, explain with Examples.

Part-IV

4. a) Use simplex method to solve the LPP 7

$$\text{Max} \quad z = 3x_1 + 2x_2$$

$$\text{Subject to constraints } x_1 + x_2 \leq 4$$
$$x_1 - x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

OR

b) Use Big M method to solve LPP

$$\text{Mini } z = 12x_1 + 20x_2,$$

$$\text{Subject to constraints } 3x_1 + 4x_2 \geq 50$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1 \geq 0, x_2 \geq 0.$$

5. a) Write the dual of the following LPP and solve. 7

Maxi $z = 8x_1 + 4x_2$,

Subject to constraints $4x_1 + 2x_2 \leq 30$

$2x_1 + 4x_2 \leq 24$

$x_1 \geq 0, x_2 \geq 0.$

OR

b) Prove that the dual of the dual is the primal.

6. a) Use Vogel's Approximation method to obtain an initial basic feasible solution of TP

	D	E	F	G	Available
A	12	23	10	20	250
B	13	22	11	17	275
C	18	15	18	14	425
Demand	225	200	250	275	

OR

b) Solve the following Assignment problem :

$$\begin{array}{c} \\ I \\ II \\ III \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix} \end{array}$$

7. a) Solve the following 2-person zero-sum-game.

Player B

$$\text{Player A}\begin{bmatrix} 15 & 2 & 3 \\ 16 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$$

7

OR

b) For the game with the following pay off matrix, determine the optimal strategies and the value of the game

P_2

$$P_1\begin{bmatrix} 5 & 3 \\ 1 & 4 \end{bmatrix}$$

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Answer *all* questions

Part-I

1. Answer the following : 1 × 12

   a) What is the probability of impossibe event ?

   b) If A and B are independent event, then is A and B′ are independent ?

   c) What is the probability of getting other than 5 in a single throw of die.

   d) Define moment-generating function of x, when x is discrete.

   e) If x and y are indpendent, the what is the value of σ_{xy}.

   f) What do you mean by co-variance of Random variable X and Y.

   g) Define gama function.

[Turn Over

h) Write the formulas of variance of chisquare distribution.

i) What is the relation between Beta and Gamma function ?

j) Write the difference between discrete and continuous distribution.

k) What do you mean by random variable ?

l) What is linear regression ?

Part-II

2. Answer any ***eight*** of the following : 2×8

a) Find the probability of sum of eight if two dice is rolled once.

b) State the Baye's Theorem.

c) If x has the probability density

$$f(x) = \begin{cases} 3e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

then find $P(0.5 \leq x \leq 1)$

d) If b is constant, then prove that $E(bx) = bE(x)$.

e) if X : are random variable and a_i are constant for i = 1 to n, and $Y_i = \sum_{i-1}^{n} a_i Xi$ then prove that $E(Y) = \sum_{i-1}^{n} a_i E(x_i)$.

f) Prove that Mean Distribution of Beta function $\mu = \frac{\alpha}{\alpha + \beta}$.

g) Find μ for the random variable X which has probability density

$$f(x) = \begin{cases} x/2 & \text{for } 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

h) If X is the number of points rolled with a balanced die, then find the expected value of $g(X) = 2X + 1$.

i) Write the formulla for sample mean and variance of finite population $\{c_1, c_2, c_3 \ldots c_N\}$.

j) What is the probability density function of chi-square distribution with ν degree of freedom.

Part-III

3. Answer any ***eight*** of the following : 3×8

a) Prove that

$$P(A' \cap B') + P(A) + P(B) = 1 + P(A \cap B).$$

b) If $f(x) = kx$ for $x = 1, 2, 3, 4, 5, 6$, is probability distribution random variable, then find the value of of K.

c) Show that $F(x) = 3x^2$ for $0 < x < 1$ is a probability density function.

d) The probability density of X is given by

$$f(x) = \begin{cases} \dfrac{1}{x \ln 3} & \text{for } 1 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

then find $E(X^2)$.

e) Find the moment-generating function of the random variable whose probability density is given by

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

f) Find the expression for mean of Binomial distribution.

g) Find the expression for the mean of poisson distribution.

h) Define and explain t-distribution.

i) Define and explain F-distribution.

j) If $\bar{X}$ is the mean of a random sample of size n taken without replacement from a finite pouplation of size N with the mean μ, then prove that $E(\overline{X})=\mu$.

Part-IV

4. a) Find the probability distribution of the total number of head obtained in four tosses of a balanced coin. 7

OR

b) Find the Marginal densities of X and Y where joint probability density is given by

$$F(x,y)=\begin{cases}\frac{1}{4}(2x+3y) \text{ for } 0<x<1, 0<y<2\\ 0 \quad \text{otherwise}\end{cases}$$

5. a) The joint probability density of X and Y are given by 7.

$$F(x,y)=\begin{cases}\frac{2}{3}(x+2y)\text{ for } 0<x<1, 0<y<1\\ 0 \quad \text{elsewhere}\end{cases}$$

Then find the conditional mean and conditional variance of X given $Y=\frac{1}{2}$. 7

OR

b) $X_1, X_2 \ldots X_n$ are random variable and

$Y_1=\sum_{i=1}^{n} a_i X_i, Y_2=\sum_{i=1}^{n} b_i X_i$, a_i and b_i are constant for i = 1 to n. Then prove that

$$Cov(Y_1 Y_2)=\sum_{i=1}^{n} a_i b_i, \ var(X_1)+\sum_{i<j}(a_i b_j + a_j b_i) cov(x_i, y_j)$$

6. a) Prove that the variance of Binomial Distribution is $\sigma^2 = n\theta(1-\theta)$. 7

OR

b) Prove that the moment -generating function of Normal distribution is $M_x(t)=e^{ut+\frac{1}{2}\sigma^2 t^2}$.

7\. a) If Xr and Xs are the rth and sth random variables of random sample of size n drawn from the finite population $\{c_1, c_2, c_3 \dots c_N\}$, then prove that

$$Cov(X_r, X_s) = -\frac{\sigma^2}{N-1}.$$ 7

OR

b) If the joint density of X_1, X_2 and X_3 is given by

$$f(x) = \begin{cases} (x_1 + x_2)e^{-x_3} & \text{for } 0<x_1<1, 0<x_2<2, x_3>0 \\ 0 & \text{elsewhere} \end{cases}$$

Then find regression equation of X_2 on X_1 and X_3.